

1. Compute $\int_{-\pi}^{2\pi} \theta^4 d\theta$

- a. $12\pi^3$ b. $\frac{33\pi^5}{5}$ c. $\frac{31\pi^5}{5}$ d. -12θ e. None of these

2. Given $f(x) = x \ln x + x$, find c if $f'(c) = 0$.

- a. $\frac{e}{4}$ b. $\frac{1}{e^2}$ c. -1 d. ln2 e. None of these

3. For which value of a is $\int_1^a (3x^2 - 6x + 3) dx = 27$

- a. 1 b. 2 c. 3 d. 4 e. None of these

4. If $\sin x = e^y$, find $\frac{dy}{dx}$ in terms of x.

- a. $\cot x$ b. $\frac{\cos x}{e}$ c. $\ln(\sin x)$ d. $\frac{1}{\ln(\sin x)}$ e. None of these

5. If $f(x) = e^x$, which of the following is equal to $f'(x)$?

- a. $\lim_{\Delta x \rightarrow 0} \frac{e^{x+\Delta x}}{\Delta x}$ b. $\lim_{\Delta x \rightarrow 0} \frac{e^{x+\Delta x} - e^x}{\Delta x}$ c. $\lim_{\Delta x \rightarrow 0} \frac{e^{x+\Delta x} - e}{\Delta x}$ d. $\lim_{\Delta x \rightarrow 0} \frac{e^{x+\Delta x} - 1}{\Delta x}$ e. $\lim_{\Delta x \rightarrow 0} e^x \frac{e^{\Delta x} - 1}{\Delta x}$

6. Let f be a continuous function on the interval $[-5, 10]$ and let $g(x) = f(x) + 2$. If

$\int_{-5}^{10} f(t) dt = 4$, what is $\int_{-5}^{10} g(u) du$?

- a. 6 b. 19 c. 22 d. 34 e. None of these

7. Let f and g have continuous first and second derivatives everywhere. If $f(x) \leq g(x)$ for all real x , which of the following must be true?

I. $f'(x) \leq g'(x)$ for all real x

II. $f''(x) \leq g''(x)$ for all real x

III. $\int_0^1 f(x) dx \leq \int_0^1 g(x) dx$ for all real x

- a. None b. I only c. III only d. I and II only e. I, II, and III

8. Which of the following is not equal to 0?

- a. $\int_{-\pi}^{\pi} \sin^3 x dx$ b. $\int_{-\pi}^{\pi} x^2 \sin x dx$ c. $\int_{-\pi}^{\pi} \cos x dx$
d. $\int_{\pi}^{\pi} \sin^3 x dx$ e. $\int_{-\pi}^{\pi} \cos^2 x dx$

9. Find the volume of the solid of revolution formed by revolving the region bounded by

$$y = \sqrt{1 - x^2} \text{ and } y = 0 \text{ about the x-axis.}$$

- a. π^2 b. $\frac{3\pi^2}{2}$ c. $\frac{5\pi^2}{12}$ d. $\frac{4\pi}{3}$ e. none of these

10. Find the value of the integral by eliminating all obviously wrong answers:

$$\int_0^{\pi} \sin^8 x dx$$

- a. π b. $\frac{35\pi}{128}$ c. $\frac{\pi}{10} - 1$ d. 0 e. $\frac{705\pi}{256}$

11. If the substitution $u = \frac{x}{2}$ is made, the integral $\int_2^4 \frac{1 - (\frac{x}{2})^2}{x} dx =$

- a. $\int_1^2 \frac{1-u^2}{u} du$ b. $\int_1^2 \frac{1-u^2}{4u} du$ c. $\int_2^4 \frac{1-u^2}{u} du$ d. $\int_2^4 \frac{1-u^2}{2u} du$

e. none of these

12. When air expands without losing or gaining heat, the volume, V in cubic centimeters and the pressure P in kilopascals (kPa) are related by the equation $PV^k = C$, where k and C are constants. When the volume is 10 cm^3 and increasing by $4 \text{ cm}^3/\text{min}$, the pressure is 25 kPa and is decreasing by 14 kPa/min . Find the value of k ?

- a. $5/2$ b. $14/25$ c. $7/5$ d. $2/25$ e. 14.25

13. The range of a projectile whose muzzle velocity in meters per second is v , and whose angle of elevation in radians is Θ , is given by $R = \frac{v^2 \sin 2\theta}{g}$ where g is the acceleration of gravity. Which angle of elevation gives the maximum range of the projectile?

- a. 27° b. 30° c. 35° d. 45° e. 60°

14. Let $f(x) = \begin{cases} 2 & \text{if } x < 0, \\ 3 - x & \text{if } 0 \leq x \leq 1, \\ x^2 + 1 & \text{if } x > 1. \end{cases}$

Which correctly describes the continuity of f at $x=0$ and $x=1$?

- a. Continuous at $x=0$ and $x=1$
- b. Continuous at $x=0$, discontinuous at $x=1$
- c. Discontinuous at $x=0$ and $x=1$
- d. Discontinuous at $x=0$, continuous at $x=1$
- e. Discontinuous at $x=2$

15. Which of the following are ALWAYS true:

- i. If $f(1) < 0$ and $f(2) > 0$, then there must be a point z in $(1,2)$ such that $f(z) = 0$.
- ii. If f is continuous on $[1,2]$, $f(1)<0$ and $f(2)>0$, then there must be a point z in $(1,2)$ such that $f(z) = 0$.
- iii. If f is continuous on $[1,2]$, and there is a point z in $(1,2)$ such that $f(z) = 0$, then $f(1)$ and $f(2)$ must have different signs.
- iv. If f has no zeros and is continuous on $[1,2]$, then $f(1)$ and $f(2)$ have the same sign.
- a. i and iv b. ii and iii c. ~~i, ii, and iv~~ d. ii and iv e. ii only

16. Consider the graph of $x^2 + y^2 = c^2$, where c is some non-zero constant. For which quadrants will $\frac{dy}{dx}$ be positive?

- a. I and II b. II and IV c. I and III d. I and IV e. II and III

17. Find $\lim_{x \rightarrow 0} \frac{e^{3+x} - e^3}{x}$

- a. e
- b. e^3
- c. 1
- d. x
- e. $\ln x$

18. If $f'(x) \leq 2$ for all real numbers in the interval $[0,3]$ and $f(0) = -1$, how large can $f(3)$ possibly be?

- a. 2
- b. 3
- c. 4
- d. 5
- e. 6

19. Let $f(x) = \left| \cos^2 x - \frac{1}{3} \right|$. Find the absolute maximum and minimum values of f .

- a. abs. max. $2/3$, abs. min. 0 b. abs. max. $5/3$, abs. min. $1/3$ c. abs. max. $2/3$, abs. min. $1/3$
d. abs. max 1 , abs. min $-1/3$ e. abs. max. $5/3$, abs. min $-1/3$

20. Let f be a continuous, differentiable function which satisfies $f(x+y) = f(x) + f(y) + 2xy$ for all real numbers x and y , and suppose $\lim_{h \rightarrow 0} \frac{f(h)}{h} = 7$. Use the definition of derivative to find $f'(x)$.

- a. $7+2x$ b. $-2xy$ c. $14x$ d. $7 - 14x$ e. $2x$

21. Suppose f is a function for which $\lim_{x \rightarrow 2} \frac{f(x)-f(2)}{x-2} = 0$. Which of the following statements MUST be true.

- i. $f'(2) = 2$. ii. $f(2) = 0$ iii. $\lim_{x \rightarrow 2} f(x) = f(2)$ iv. $f(x)$ is continuous at $x = 0$
a. i and iv b. ii c. iii d. ii and iv e. ii and iii

22. Let $f(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ \pi & \text{if } x = 0. \end{cases}$ Find $\lim_{x \rightarrow 0} f(f(x))$.

- a. 1 b. -1 c. 0 d. π e. Does not exist

23. Find a formula for the n th derivative of the function, $f(x) = \frac{1}{x}$. Hint: take the first 3 derivatives and find a pattern. $f^{(n)}(x) =$

- a. $\frac{x^n}{n!}$ b. $\frac{(-1)^n n!}{x^{n+1}}$ c. $\frac{(-1)^{n+1} n!}{x^n}$ d. $-\frac{n^2}{x^n}$ e. $-x^{-n}$

24. Find $\frac{dy}{dx}$ for the curve: $(x^2 + y^2)^2 = 12(x^2 - y^2)$.

- a. 1 b. $\frac{x-x^2-y^2}{y}$ c. $\frac{12x-x^2-y^2}{12y}$ d. $\frac{6(x^2+y^2)}{(x^2-y^2)}$ e. $\frac{x}{y} \cdot \frac{6-(x^2+y^2)}{6+(x^2+y^2)}$

25. Where is the function, $f(x) = (1000 - x)^2 + x^2$, increasing?

- a. $(500, \infty)$ b. $(-2000, \infty)$ c. $(100, 250)$ d. $(-\infty, -250)$ e. never increases

26. Use the table below to find $D_x[(f(g(x)))]$ for $x = 3$

x	0	1	2	3	4
$f(x)$	$\frac{1}{2}$	$\frac{1}{3}$	1	-1	3
$g(x)$	-2	1	$-\frac{1}{2}$	2	$-\frac{1}{3}$
$f'(x)$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{1}{4}$	0	$-\frac{4}{5}$
$g'(x)$	-1	$\frac{2}{3}$	-4	-3	$-\frac{1}{3}$

- a. $2/5$ b. $1/3$ c. 0 d. $-8/5$ e. $-3/4$

27. Find intervals over which $f(x) = \left| x^2 - \frac{6}{25} \right|$ is increasing.

- a. $\left(\frac{-3}{5}, 0 \right) \cup \left(\frac{3}{5}, \infty \right)$ b. $\left(\frac{\sqrt{6}}{5}, \infty \right)$ c. $(-\infty, 0)$ d. $\left(\frac{-\sqrt{6}}{5}, 0 \right) \cup \left(\frac{\sqrt{6}}{5}, \infty \right)$ e. $(-\infty, \infty)$

28. Find the derivative of $f(x) = \cos(\cos(x^2 + 1))$.

a. $-\sin^2(2x)$ b. $2x\sin(x^2 + 1)\sin(\cos(x^2 + 1))$ c. $4x\cos(x^2 + 1)$

d. $2x\sin^2(\cos(x^2 + 1))$ e. $2x\cos(\sin^2(x^2 + 1))\cos(\sin(x^2 + 1))$

29. Find b such that the line $y = b$ splits in half the region bounded by $y = 4$ and $y = x^2$.

a. 2 b. $4^{\frac{2}{3}}$ c. $2^{\frac{5}{4}}$ d. $\frac{8}{3}$ e. $2^{\frac{3}{2}}$

30. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^1 (y\cos x + 2) dy dx$

a. $\frac{3\pi}{2} - 1$ b. -1 c. $\frac{1}{2} + \pi$ d. $\pi - \frac{1}{2}$ e. $\frac{\pi}{2} - 1$

31. Determine whether $\sum_{k=0}^{\infty} \frac{3^{2k+1}}{10^k}$ converges or diverges, and if it converges find what it converges to.

a. converges to 30 b. converges to $\frac{10}{7}$ c. converges to $\frac{123}{10}$ d. converges to 93 e. Diverges

32. Find the largest possible value of the function of two variables $z = f(x, y) = 2x + 4y - x^2 - y^2$

a. 6 b. $\frac{1}{3}$ c. 8 d. 5 e. 60

33. Find the slope of the surface $g(x, y) = 4 - x^2 - y^2$ in the x -direction at the point $(1, 1, 2)$.

- a. 4 b. 1 c. -4 d. -2 e. 0

34. For the function $f(x, y) = \frac{1}{x} + \frac{1}{y} + xy$, find all values of x and y such that $f_x(x, y) = 0$ and $f_y(x, y) = 0$.

- a. $(1, 1)$ and $(-1, -1)$ b. $(1, 1)$ c. $(-1, -1)$ and $(0, 0)$ d. $(0, 0)$ and $(1, 1)$ e. $(-1, -1)$

35. Given $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$, find $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 + \frac{2i}{n}\right)^2 \left(\frac{2}{n}\right)$

- a. $\frac{77}{6}$ b. $\frac{39}{4}$ c. 8 d. $\frac{35}{6}$ e. $\frac{26}{3}$

36. Calculate $\int_{-3}^3 (x + 5)\sqrt{9 - x^2} dx$.

- a. $\frac{45}{2}\pi$ b. $\frac{-4\sqrt{3}+45\pi}{2}$ c. 0 d. 90π e. $\frac{-2\sqrt{3}+90\pi}{3}$

37. Find the volume of the solid of revolution generated by revolving the region bounded by

$y = \sqrt{x}$, $y = 0$, and $x = 4$ about the line $x = 6$.

- a. $\frac{200\pi}{3}$ b. $\frac{158\pi}{3}$ c. 56π d. $\frac{14\pi}{3}$ e. None of these

38. Evaluate the limit: $\lim_{x \rightarrow 1} \frac{1+x(\ln x - 1)}{(x-1)\ln x}$

- a. 1 b. -1 c. ∞ d. $-\infty$ e. $\frac{1}{2}$

39. Find $\frac{d^2y}{dx^2}$ for $2x + y^2 = xy - 1$.

- a. $\frac{-10}{(2y-x)^3}$ b. $\frac{2y^2-2xy+4x-6}{(2y-x)^2}$ c. $\frac{2y^2-2xy-4x-6}{(2y+x)^2}$ d. $\frac{-6}{(2y-x)^5}$ e. $\frac{2}{(2y-x)^2}$

40. Evaluate: $\int_{-2}^2 \frac{2x^5 - x^3 + 4x}{x^3 + 8x} dx$.

- a. $\frac{1}{2}$ b. $\frac{5}{8}$ c. 0 d. diverges e. $\frac{3}{4}$

Key for AlaMATYC Calculus Test 2020

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| 1. B | 21. C |
| 2. B | 22. D |
| 3. D | 23. B |
| 4. A | 24. E |
| 5. E | 25. A |
| 6. D | 26. E |
| 7. C | 27. D |
| 8. E | 28. B |
| 9. D | 29. B |
| 10. B | 30. C |
| 11. A | 31. A |
| 12. C | 32. D |
| 13. D | 33. D |
| 14. D | 34. B |
| 15. D | 35. E |
| 16. B | 36. A |
| 17. B | 37. C |
| 18. D | 38. E |
| 19. A | 39. A |
| 20. A | 40. C |