



The **Alabama**
Mathematical Association of Two Year Colleges

Calculus Morning Competition Key

Good morning!

Please do NOT open this booklet until given the signal to begin.

There are 40 multiple choice questions and you will be given 90 minutes to complete the test. Record your answers on the electronic grading form by giving the best answer to each question.

The scoring will be done by giving one point for each question answered correctly and zero points for each question answered incorrectly or left blank. Thus, it is to your advantage to answer as many questions as possible, even if you have to guess. If there is a tie, we will start at question number 40 and work backwards until the tie is resolved.

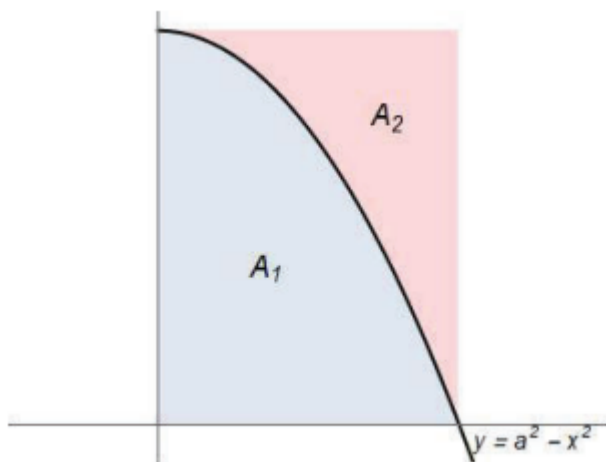
This test was designed to be a CHALLENGE. Do not waste time on questions you are unable to answer; focus and take pride in those questions which you ARE able to answer.

You may write in the test booklet. You may keep your test booklet and any of your scrap papers. Only the electronic grading form will be collected and graded.

Good luck!

Do Not Open Until Signaled.

1. Consider the parabola $y = a^2 - x^2$ where a is a real number. An image of such a parabola is shown below, along with the region enclosing A_1 and A_2 units of area respectively. For which real numbers a , if any, is A_1 twice as large as A_2 ?



- A. $a = 1, -1$
 B. $a = \sqrt{3}, -\sqrt{3}$
 C. $a = 0$
 D. $a = \text{all real numbers}$
 E. None of the above
2. Find the numbers, a , such that $\lim_{x \rightarrow 0} \frac{e^{ax^2} - \cos(2x)}{x^2} = 8$.
- A. 6 B. 2 C. 3 D. 4 E. None of the above
3. Find the values of the constants a and b such $f(x)$ is differentiable everywhere.

$$f(x) = \begin{cases} ax^3 + bx + 2 & x \leq 2 \\ bx^2 - a & x > 2 \end{cases}$$

- A. $a = -2, b = 10$
 B. $a = -2, b = -8$
 C. $a = 2, b = 10$
 D. $a = 2, b = 8$
 E. None of the above

4. If $f'(x) = h(x)$ and $g(x) = x^4 - 2$, then find $\frac{d}{dx}[(f \circ g)(x)]$.
- A. $h(x^4 - 2)$
 B. $4x^3h(x^4 - 2)$
 C. $4x^3h(x)$
 D. $4x^3h'(x^4 - 2)$
 E. None of the above
5. Given that $\int_0^2 f(x) dx = \frac{8}{3}$, $\int_1^2 f(x) dx = \frac{4}{3}$, and $\int_0^3 f(x) dx = \frac{11}{3}$, find $\int_3^1 f(x) dx$.
- A. $\frac{7}{3}$ B. $\frac{5}{3}$ C. $-\frac{5}{3}$ D. $-\frac{7}{3}$ E. None of the above
6. The function $f(x) = x^3 + 2x - 9$ has an inverse. If the graph of f passes through the point $(2, 3)$, then find $(f^{-1})'(3)$.
- A. $\frac{1}{12}$ B. $\frac{1}{14}$ C. $\frac{1}{29}$ D. $\frac{1}{2}$ E. None of the above
7. What is $\lim_{h \rightarrow 0} \frac{\cos(\frac{5\pi}{6} + h) - \cos(\frac{5\pi}{6})}{h}$?
- A. $\frac{\sqrt{3}}{2}$ B. $-\frac{\sqrt{3}}{2}$ C. $\frac{1}{2}$ D. $-\frac{1}{2}$ E. None of the above
8. Find an equation for the normal line to the curve $2x^3 + 2y^2 = 5xy$ at the point $(1, 2)$.
- A. $3x + 4y = 11$
 B. $4x - 3y = -2$
 C. $4x + 3y = 10$
 D. $3x - 4y = -5$
 E. None of the above
9. What is the maximum value of $f(x) = \frac{\ln(x)}{x}$?
- A. e^{-1} B. e C. 1 D. $\sqrt{e}$ E. None of the above

10. Which of the following represents the area of the region inside the circle $r = 3\sin(\theta)$ and outside the cardioid $r = 1 + \sin(\theta)$?

A. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2\sin(\theta) - 1)^2 d\theta$
B. $\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (8\sin^2(\theta) - 1) d\theta$
C. $\frac{9\pi}{4} - \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (1 + \sin(\theta))^2 d\theta$
D. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} [9\sin^2(\theta) - (1 + \sin(\theta))^2] d\theta$
E. None of the above

11. Find the area of the region in the first quadrant ($x > 0, y > 0$) bounded above the graph $y = \arcsin(x)$ and below the graph $y = \arccos(x)$.

A. $\sqrt{2} - 1$ B. $\sqrt{2} + 1$ C. $2 + \sqrt{2}$ D. $2 - \sqrt{2}$ E. None of the above

12. Find the derivative of $f(x) = x^x$.

A. x^{x-1} B. x^x C. $x^x(\ln(x) + 1)$ D. $x^x(\log(x) + 1)$ E. None of the above

13. Let $f(x)$ be a continuous function on the interval $[-3, 7]$ and let $g(x) = 8f(x) - 3$. If $\int_{-3}^7 f(t) dt = 6$, find $\int_{-3}^7 g(u) du$.

A. 78 B. 18 C. 36 D. 60 E. None of the above

14. Let $f(x) = \frac{2}{3}x^3 - 6x + 12$. Find the point(s) where $f(x)$, $f'(x)$, and $f''(x)$ all coincide.

A. $(-1, \frac{52}{3})$ B. $(-3, 12)$ C. $(1, \frac{20}{3})$ D. $(3, 12)$ E. None of the above

15. Find the limit. $\lim_{x \rightarrow \infty} \frac{\cos(6x)}{x^2}$

A. 6 B. 0 C. 1 D. -1 E. None of the above

16. Suppose $f(x)$ is an odd function and $g(x)$ is an odd function. If $h(x) = f(x)g(x)$, which of the following is/are equivalent to $\int_{-a}^a h(x) dx$?

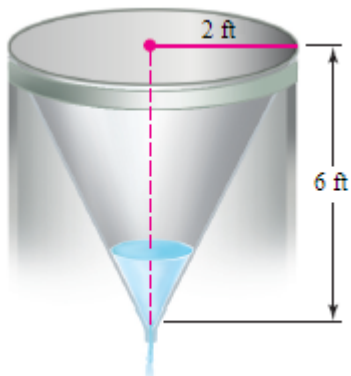
1. $2\int_{-a}^0 h(x) dx$
2. $-\int_{-a}^a h(-x) dx$
3. 0
4. $\int_{-a}^a h(-x) dx$

A. 1 and 4 B. 3 only C. 2 and 3 D. 1 only E. None of the above

17. Find the equation of the tangent line to the cissoid of Diocles, $y^2(10 - x) = 4x^3$ at the point $(5, 10)$.

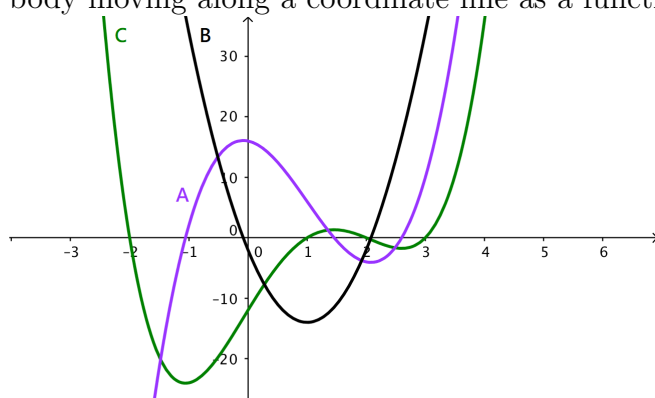
- A. $y = 4x - 10$
- B. $y = \frac{4}{3}x + \frac{10}{3}$
- C. $y = 8x - 30$
- D. $y = \frac{8}{3}x - \frac{10}{3}$
- E. None of the above

18. Water is drained out of an inverted cone, having dimensions depicted in the figure below. If the height of the water drops at a rate of 4 feet per minute, at what rate is the volume of water changing at the moment the depth of the water is 3 feet?



- A. $-4\pi \frac{ft^3}{min}$
- B. $-2\pi \frac{ft^3}{min}$
- C. $-8\pi \frac{ft^3}{min}$
- D. $-12\pi \frac{ft^3}{min}$
- E. None of the above

19. The graphs in the figure below show the position, s , velocity, v , and acceleration, a of a body moving along a coordinate line as a function of time t . Which graph is which?



- A. The graph labeled B is the graph of the position, s .
The graph labeled A is the graph of the velocity, v .
The graph labeled C is the graph of the acceleration, a
- B. The graph labeled A is the graph of the position, s .
The graph labeled B is the graph of the velocity, v .
The graph labeled C is the graph of the acceleration, a
- C. The graph labeled B is the graph of the position, s .
The graph labeled C is the graph of the velocity, v .
The graph labeled A is the graph of the acceleration, a
- D. The graph labeled C is the graph of the position, s .
The graph labeled A is the graph of the velocity, v .
The graph labeled B is the graph of the acceleration, a .
- E. None of the above

20. If $f(x) = (x - 2)^3(x - 3)^2$, find $f''(2) + f'(3)$.

- A. 0 B. -2 C. 1 D. 2 E. None of the above

21. Evaluate $\int_{2^5}^{3^5} \frac{1}{x - x^{\frac{3}{5}}} dx$.

- A. $\ln(2)$ B. $\frac{5}{2}\ln(2)$ C. $\ln(\frac{8}{3})$ D. $\frac{5}{2}\ln(\frac{8}{3})$ E. None of the above

22. Evaluate $\int_0^4 e^{\sqrt{x}} dx$.

- A. $2e^2 + 2$ B. $2e^{\sqrt{2}} - 2$ C. $2e^2 - 2$ D. $2e^{\sqrt{2}} + 2$ E. None of the above

23. Find the limit. $\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 - x}}$

- A. ∞ B. -1 C. 1 D. $-\infty$ E. None of the above

24. Evaluate $\int_{-4}^4 \frac{dx}{16+x^2}$.

- A. $\frac{\pi}{8}$ B. $\frac{\pi}{4}$ C. 0 D. $\frac{\pi}{2}$ E. None of the above

25. Find the instantaneous rate of change of the curve at the point defined by the given values of t .

$$x = 3t, y = \sqrt{t}, t = \frac{1}{4}$$

- A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{3}{4}$ D. 1 E. None of the above

26. Find the equation of the tangent plane to the surface $x^2 - 3y^2 - 4z^2 = 2$ at the point $(3, 1, 1)$.

- A. $3x + 3y - 4z = -2$
B. $3x - 3y - 4z = 2$
C. $-3x - 3y + 4z = 2$
D. $3x + 3y + 4z = -2$
E. None of the above

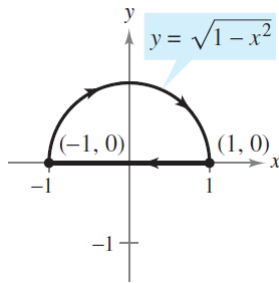
27. Evaluate $\int_0^1 \int_1^{e^y} \frac{y}{x} dx dy$.

- A. $-\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$ E. None of the above

28. Find $\int_C y \, ds$ if C is the curve defined by the parametric equations $x(t) = t$ and $y(t) = \sqrt{t}$ for $2 \leq t \leq 6$.

- A. $-\frac{49}{3}$ B. $\frac{38}{3}$ C. $\frac{77}{6}$ D. $\frac{49}{6}$ E. None of the above

29. Find the work done by the force field, $F = 3xi + 2yj$ along the path shown below.



- A. $\frac{\pi}{2}$ B. π C. 2π D. 0 E. None of the above

30. What quadric surface is represented by the equation $15x^2 - 4y^2 + 15z^2 = -4$?

- A. Hyperboloid of 1 sheet
 B. Ellipsoid
 C. Elliptic Cone
 D. Hyperboloid of 2 sheets
 E. None of the above

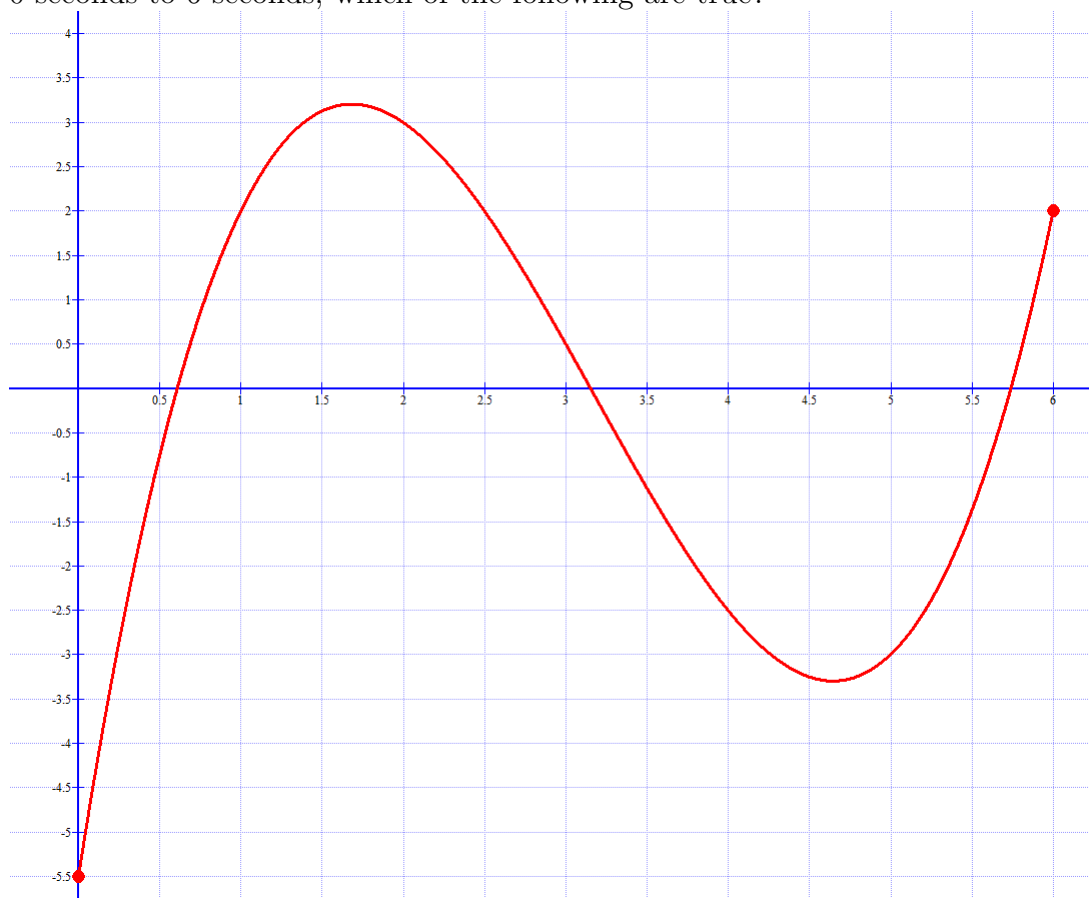
31. Find the limit. $\lim_{x \rightarrow 2} \frac{\frac{5}{x^2 - 10x + 16}}{\frac{4}{x-2} - \frac{3}{x-8}}$

- A. $\frac{5}{14}$ B. $-\frac{5}{28}$ C. 0 D. $-\frac{5}{24}$ E. None of the above

32. Find the derivative. $f(x) = \ln\left(\frac{e^x(x-3)^2}{x^2-4}\right)$.

- A. $\frac{x^2-4}{e^x(x-3)^2}$
 B. $e^x - \frac{2}{x-3} + \frac{2x}{x^2-4}$
 C. $1 + \frac{2}{x-3} - \frac{2x}{x^2-4}$
 D. $e^x + \frac{1}{x-3} + \frac{1}{x+2}$
 E. None of the above

33. If the graph below is a graph of the position function of an object over the interval from 0 seconds to 6 seconds, which of the following are true?



- i. The object is slowing down on the interval from 2 seconds to 3 seconds.
- ii. The object is slowing down on the interval from 3.5 seconds to 4.5 seconds.
- iii. The displacement of the object is 7.5 units.
- iv. The object traveled a total distance of 7.5 units.

- A. i and iv only
- B. ii and iii only
- C. i only
- D. i, ii, and iv only
- E. None of the above

34. Evaluate $\int \frac{x^3+4x^2-3x-1}{x-2} dx$.

- A. $\frac{x^3}{3} + x^2 - 7x + 13\ln|x-2| + C$
- B. $\frac{(x^3+4x^2-3x-12)^2}{2(x-2)} + C$
- C. $\frac{x^3}{3} - x^2 - x - 9\ln|x-2| + C$
- D. $\frac{x^3}{3} + 3x^2 + 9x + 17\ln|x-2| + C$
- E. None of the above

35. If a positive number is added to twice its reciprocal, what is the minimum value?

- A. $2\sqrt{2}$ B. $3\sqrt{3}$ C. 1 D. $\frac{1}{4}$ E. None of the above

36. Find the limit. $\lim_{x \rightarrow 0} (\ln|x| - \ln|\sin(x)|)$.

- A. e B. 0 C. 1 D. π E. None of the above

37. Find the average value of $f(x) = \sin(x)\cos^4(x)$ over the interval $[0, \pi]$.

- A. $\frac{2}{5}$ B. $\frac{1}{5}$ C. $\frac{2}{5\pi}$ D. $\frac{1}{5\pi}$ E. None of the above

38. Suppose $f(x)$ is a differentiable function such that $f'(x) \leq 3$ for all $x \in [-3, 4]$. If $f(-3) = 4$, what is the largest possible value of $f(4)$.

- A. -15 B. 10 C. 25 D. 60 E. None of the above

39. Evaluate $\int t(t+2)^5 dt$.

- A. $\frac{t^7}{7} + \frac{t^6}{3} + C$
B. $\frac{t(t+2)^6}{6} + C$
C. $\frac{(t+2)^7}{7} + \frac{(t+2)^6}{3} + C$
D. $\frac{t^2(t+2)^7}{12}$
E. None of the above

40. Define $f(3)$ in a way that extends $f(x) = \frac{x^2-9}{x^2-5x+6}$ to be continuous at $x = 3$.

- A. 6 B. 0 C. 3 D. 4 E. None of the above