The Alabama Mathematical Association of Two Year Colleges

Team Competition #1

How many squares are there on a standard 8 x 8 checkerboard?

(Hint: not all of them are 1 x 1)

Find a pattern:

In a 1 x 1 checkerboard there would be only one square

1

In a 2 x 2 checkerboard there would be 4 small squares(1x1) and the 1 large square(2x2)

5

In a 3 x 3 checkerboard there would be 9 small squares (1x1), 4 squares(2x2), and 1 square (3x3)

14

Continuing with this we see a pattern

1 x 1 had 1^2

2 x 2 had 1^2 + 2^2

3 x 3 had 1^2 + 2^2 + 3^2

So, and 8 x 8 has $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 = 204$
(1, 4, 9, 16, 25, 36, 49, 64)

Answer:

204

Time on Clock:

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Team Competition #2

Find a right triangle with integral side lengths and area 84 square units.

Using x and y for the legs we have ½ *x*y = 84, so xy=168, and hypotenuse is $\sqrt{x^2 + y^2}$

Factors of 168

1, 168

2, 84

3, 56

4, 42

6, 28

7, 24... note that 7, 24, and 25 form a Pythagorean triple, so these are the sides

Answer:

7, 24, 25

Time on Clock:

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Team Competition #3

Solve: $x^{\sqrt[3]{\log_5 x}} = 5^{16}$

$\log_5 x^{\sqrt[3]{\log_5 x}} = \log_5 5^{16}$

$\sqrt[3]{\log_5 x}\,(\log_5 x)^1 = 16$

$(\log_5 x)^{4/3} = 16$

$\log_5 x = 16^{3/4}$

$\log_5 x = 8$

$5^{\log_5 x} = 5^8$

$x = 390{,}625$

Answer:

390,625 or 5^8

Time on Clock:

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Team Competition #4

Given that $2-5i$ is a zero of $f(x)=x^4-12x^3+66x^2-252x+145$, find the other zeros. Give exact answers.

Zeros: $2-5i$, $2+5i$, ____, ____

$(x-2+5i)(x-2-5i)$

$x^2-2x-5ix-2x+4+10i+5ix-10i-25i^2$

$x^2-4x+29$

$$\begin{array}{r} x^2-8x+5 \\ x^2-4x+29\overline{)\,x^4-12x^3+66x^2-252x+145} \\ x^4-4x^3+29x^2 \\ \hline -8x^3+37x^2-252x+145 \\ -8x^3+32x^2-232x \\ \hline 5x^2-20x+145 \\ 5x^2-20x+145 \\ \hline 0 \end{array}$$

$x^2-8x+5=0$

$x=\dfrac{8\pm\sqrt{64-4(1)(5)}}{2(1)}$

$x=\dfrac{8\pm\sqrt{44}}{2}=\dfrac{8\pm2\sqrt{11}}{2}$

$=4\pm\sqrt{11}$

Answer:

$2+5i,\ 4+\sqrt{11},\ 4-\sqrt{11}$

Time on Clock:

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Team Competition #5

Find the domain of $f(x) = \frac{\sqrt{3x^2+x-10}}{x^2-4}$. Give answer in interval notation.

$g(x) = 3x^2 + x - 10 > 0$

$3x^2 + x - 10 = 0$

$(3x-5)(x+2) = 0$

$x = \frac{5}{3} \quad x = -2$

$x^2 - 4 \neq 0$

$x \neq \pm 2$

$g(-3)$ $\quad g(0) < 0 \quad$ $g(2)$

$27 - 3 - 10 > 0$ $\qquad 12 + 2 - 10 > 0$

$(-\infty, -2) \cup \left[\frac{5}{3}, 2\right) \cup (2, \infty)$

Answer:

$(-\infty, -2) \cup \left[\frac{5}{3}, 2\right) \cup (2, \infty)$

Time on Clock:

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Team Competition #6

Write the equation of the vertical parabola that passes through (- 8, 5/2) and whose vertex is the center of the circle given by $2x^2 + 8x + 2y^2 - 14y + 10 = 0$

$2(x^2+4x+4) + 2(y^2 - 7y + \frac{49}{4}) = -5 + 4 + \frac{49}{4}$

$(x+2)^2 + (y-\frac{7}{2})^2 =$

vertex is $(-2, \frac{7}{2})$ & passes through $(-8, \frac{5}{2})$

$y = a(x-h)^2 + k$

$\frac{5}{2} = a(-8+2)^2 + \frac{7}{2}$

$-1 = a \cdot 36$

$-\frac{1}{36} = a$

$y = -\frac{1}{36}(x+2)^2 + \frac{7}{2}$

$(x+2)^2$
x^2+4x+4

$-\frac{1}{36}x^2 - \frac{1}{9}x - \frac{1}{9} + \frac{7}{2}$ $\quad -\frac{2}{18} + \frac{63}{18}$

Answer:

$y = -\frac{1}{36}(x+2)^2 + \frac{7}{2}$

$y = -\frac{1}{36}x^2 - \frac{1}{9}x + \frac{61}{18}$

Time on Clock:

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Team Competition #7

Given $f(-7) = 4$, $f(2) = 1$, $f(3) = 8$, and some points on $g(x)$ are (2,7), (3,-3), (5,1),

find $\dfrac{(g^{-1} \circ f)(2)}{(fg)(3)}$.

$$\frac{g^{-1}(f(2))}{f(3)\,g(3)} = \frac{g^{-1}(1)}{8 \cdot (-3)} = \frac{5}{-24}$$

Answer:

$$-\frac{5}{24}$$

Time on Clock:

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Team Competition #8

John can floor a tiny house in 4 hours. Sherry can floor a tiny house in 5 hours. When working together, John can only work at 60% of his normal speed and Sherry can only work at 80% of her normal speed due to socializing. How long will it take them, to the nearest minute, to floor the tiny house? Please include units.

John $\frac{1}{4}$ house/hr
Sherry $\frac{1}{5}$ house/hr

X = # hours to complete if working together

$$.60\left(\frac{1}{4}\right) + .80\left(\frac{1}{5}\right) = \frac{1}{x}$$

$$\frac{3}{5}\cdot\frac{1}{4} + \frac{4}{5}\cdot\frac{1}{5} = \frac{1}{x}$$

$$\frac{3}{20} + \frac{4}{25} = \frac{1}{x}$$

$$\frac{15+16}{100} = \frac{1}{x}$$

$$\frac{31}{100} = \frac{1}{x}$$

$$x = \frac{100}{31} = 3\frac{7}{31} \text{ hrs.}$$

$\frac{7}{31}\cdot 60 = \frac{420}{31}$

$31\overline{)420}$ R 17, $13\frac{17}{31}$

Answer:

Accept $\frac{100}{31}$ hour

3hr. 14 min.

194 min

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Team Competition #9

The following data was gathered relating the number of feet a car travels once the brakes are applied, y, and the number of seconds the car is in motion after the brakes are applied, x.

x =1 second, y = 46 feet

x = 2 seconds, y = 84 feet

x = 3 seconds, y = 114 feet

Find a quadratic function to model this data, and use that function to find the value of y when x = 6.

$a(1)^2 + b(1) + c = 46$ I $a+b+c=46$

$a(2)^2 + b(2) + c = 84$ II $4a+2b+c=84$

$a(3)^2 + b(3) + c = 114$ III $9a+3b+c=114$

$-I+II$

$3a+b=38$

$-I+III$

$8a+2b=68$

$-6a-2b=-76$ (×−2)

$2a=-8$

$a=-4$

$3(-4)+b=38$

$b=50$

$-4+50+c=46$

$c=0$

$f(x) = -4x^2 + 50x$

$f(6) = -4(36) + 300$

$= -144 + 300$

$= 156$

Answer:

156 ft.

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Team Competition #10

What is the probability that a number chosen at random from the natural numbers from 1 through 1000 inclusive, does not have a 2, 3, 4, or 5 as a factor?

500 are odd (not multiples of 2)

200 are multiples of 5, but 100 of these have already been taken out because they were even, so subtract out the 100 odd multiples of 5

333 are multiples of 3, but 166 of these have already been taken out since they were even, and 33 of the remaining 167 were taken out as odd multiples of 15, so subtract 134 multiples of 3

(500-100-134)/1000

=266/1000 = 133/500

Answer:

$\frac{133}{500}$ OR 26.6% OR .266

Time on Clock: