2019 AlaMATYC Math Tournament **March 2, 2019**

The Alabama Mathematical Association of Two Year Colleges

Morning Competition

Good morning!

Please do NOT open this booklet until given the signal to begin.

There are 40 multiple choice questions and you will be given 90 minutes to complete the test. Record your answers on the electronic grading form by giving the best answer to each question.

The scoring will be done by giving one point for each question answered correctly and zero points for each question answered incorrectly or left blank. Thus, it is to your advantage to answer as many questions as possible, even if you have to guess. If there is a tie, we will start at question number 40 and work backwards until the tie is resolved.

This test was designed to be a CHALLENGE. Do not waste time on questions you are unable to answer; focus and take pride in those questions which you ARE able to answer.

You may write in the test booklet. You may keep your test booklet and any of your scrap papers. Only the electronic grading form will be collected and graded.

Good luck!

Do Not Open Until Signaled.

1. Evaluate $\lim_{h\to 0} \frac{-ln(x)+ln(x+h)}{h}$

   A. $ln(x)$ B. $\frac{1}{x}$ C. e^x D. 1

2. $\int \frac{e^x}{1+e^x} dx$

   A. $e^x + x + C$ B. $\frac{e^x}{x+e^x} + C$ C. $ln|1+e^x| + C$ D. $\frac{e^x}{1+e^x} + C$

3. Find the area of the region enclosed by the curves $y = \sqrt[4]{x}$ and $y = x^4$.

   A. 1 B. $\frac{3}{5}$ C. $-\frac{15}{4}$ D. $\frac{21}{20}$

4. Find the derivative of $\frac{e^x ln(\sqrt{x})}{\sqrt[3]{x^2}}$.

   A. $\frac{e^x(xln(x)+6-ln(x))}{x^{\frac{4}{3}}}$ B. $\frac{e^x(6xln(x)+9-6ln(x))}{x^{\frac{5}{3}}}$ C. $\frac{-e^x(2xln(x)+3-3ln(x))}{6x^{\frac{4}{3}}}$ D. $\frac{e^x(3xln(x)+3-2ln(x))}{6x^{\frac{5}{3}}}$

5. Evaluate $\lim_{x\to e} \frac{e^e - e^x}{ln|x|-1}$

   A. $-e^{e+2}$ B. $-e^{e+1}$ C. $-e^{e-2}$ D. $-e^{e-1}$

6. Given that $\int_a^b f(x)dx = 3$ and $\int_b^a g(x)dx = 7$, find $\int_a^b (-4g(x) + 2f(x))dx$

   A. 34 B. -22 C. -34 D. 22

7. Find the product $\int_1^e \frac{ln(x)}{x} dx * \int_2^{\sqrt[3]{8}} \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}} dx$

   A. $\frac{1}{2}$ B. 1 C. $\frac{e^2}{2} - 1$ D. 0

8. Find $\frac{d}{dx}(\frac{f'(x)}{f(x)})$ if $f(x) = \frac{1}{cos(x)} + \frac{sin(x)}{cos(x)}$

   A. $csc(x)cot(x)$ B. $sin(x)cos(x)$ C. $sec(x)tan(x)$ D. $sec(x)csc(x)$

9. Suppose f and g are differentiable, and that $f(x) = \sqrt{x} * g(x)$. If $g(4) = 20\pi$ and $g'(4) = -\pi$, then find $f'(4)$.

   A. 3π B. 18π C. 78π D. 7π

10. Find $\frac{d^2y}{dx^2}$ if $x^3 + y^2 = 8$.

    A. $\frac{-12xy^2-9x^4y}{4y^2}$ B. $\frac{-12xy^2-9x^4}{4y^3}$ C. $\frac{-3x^2}{2y}$ D. $\frac{-12xy-9x^4}{4y^2}$

11. Find the minimum value of $f(x) = cos^4(x) + sin^4(x)$.

A. $-\frac{1}{2}$ B. 0 C. -1 D. $\frac{1}{2}$

12. What is the maximum possible area of a rectangle in which its perimeter is equal to half of its area?

A. 100 B. 144 C. 36 D. 64

13. Given the table below, find $D_x[(g \circ f)(x)]$ at x = 1.

x	1	2	3	4
$f(x)$	2	4	1	3
$f'(x)$	-6	-7	-8	-9
$g(x)$	2	3	4	1
$g'(x)$	2/7	3/7	4/7	5/7

A. $-\frac{12}{7}$ B. $-\frac{21}{7}$ C. $-\frac{18}{7}$ D. $-\frac{9}{7}$

14. A 17 foot ladder is placed against a building. The base of the ladder is slipping away from the building at a rate of 9 feet per minute. At what speed is the top of the ladder sliding down the building at the instant when the bottom of the ladder is 8 feet from the base of the bulding?

A. $\frac{24}{5}$ B. $\frac{27}{5}$ C. $\frac{17}{5}$ D. $\frac{6}{5}$

15. Evaluate $\lim_{x \to -5} \frac{\frac{3}{x+5} + \frac{9}{x+7}}{\frac{2x+11}{x^2+12x+35}}$

A. -4 B. 16 C. 6 D. Does Not Exist

16. Find the volume of the solid obtained by rotating the region bounded by the points $(0, -2), (2, 1), (0, 10)$ and the y-axis about the y-axis.

A. 32π B. 24π C. 12π D. 16π

17. Find the values of the constants a and b that result in $f(x)$ being differentiable and continuous on the interval $(0, \infty)$

$$f(x) = \begin{cases} -6ln(x)e^x & 0 < x < 1 \\ ax + bx^2 & x \geq 1 \end{cases}$$

A. $a = -6e$, $b = 6e$
B. $a = 6e$, $b = -6e$
C. $a = ln(6)$, $b = -ln(6)$
D. $a = -ln(6)$, $b = ln(6)$

18. Find the x value(s) where the slope of the tangent line of $f(x) = 2x^3 - 15x^2 - 14x + \log_3(82)$ is equal to 22.

A. $-2, -3$ B. -2 C. $-1, 6$ D. -6

19. Find $\int_2^9 |x^2 - 16| dx$

A. $\frac{385}{3}$ B. 155 C. $\frac{850}{3}$ D. $\frac{80}{3}$

20. Find: $\int (xln(x+5) + 4ln(x+5) + xln(x+3) + 4ln(x+3))dx$. Assume that $x > -3$.

A. $-\frac{1}{2}(x^2 + 8x + 15)(ln(x+3) - ln(x+5) + 1) + C$
B. $\frac{1}{2}(x^2 + 8x + 15)(ln(x+3) + ln(x+5) + 1) + C$
C. $-\frac{1}{2}(x^2 + 8x + 15)(ln(x+3) - ln(x+5) - 1) + C$
D. $\frac{1}{2}(x^2 + 8x + 15)(ln(x+3) + ln(x+5) - 1) + C$

21. Find a polynomial function of degree 4 with leading coefficient 1 that has inflection points at $x = 2, -1$, critical points at $x = \frac{3}{2}, \sqrt{6}, -\sqrt{6}$, and an initial value of -8.

A. $x^4 - 2x^3 - 12x^2 + 36x - 8$
B. $x^4 + 2x^3 + 12x^2 - 36x - 8$
C. $x^4 + 12x^3 - 2x^2 - 36x - 8$
D. $x^4 - 12x^3 + 2x^2 + 36x - 8$

22. Find $\int_{\frac{-3\pi}{4}}^{\frac{3\pi}{4}} \frac{\sqrt{3}}{2} sin(x^3)cos(x)dx$.

A. 1 B. $\sqrt{2}$ C. $\frac{\sqrt{2}}{2}$ D. 0

23. Find $\int 2x^3 cos(x^2)dx$.

A. $x^2 sin(x^2) + cos(x^2) + C$
B. $x^2 + C$
C. $-\frac{x^4}{2} sin(\frac{x^3}{3}) + C$
D. $-x^2 cos(x^2) - sin(x^2) + C$

24. Find the point of the removable discontinuity of the function $f(x) = \frac{x^2+13x-30}{x^2+24x+135}$.

A. $(-10, 12)$ B. $(-2, -\frac{4}{7})$ C. $(-15, \frac{17}{6})$ D. $(-3, -\frac{5}{6})$

25. Find an equation of the tangent line to the graph $2x^2 + 3y^2 = 165$ at the point $(9, 1)$.

A. $y = -6x + 55$ B. $y = -6x - 53$ C. $y = -\frac{1}{6}x + \frac{5}{2}$ D. $y = -\frac{1}{6}x - \frac{3}{2}$

26. Determine the interval where $f(x) = 4cos(x) - x^2$ is concave up in the interval $(0, 2\pi)$.
A. $(0, \frac{2\pi}{3}) \cup (\frac{4\pi}{3}, 2\pi)$ B. $(\frac{2\pi}{3}, \frac{4\pi}{3})$ C. $(\frac{4\pi}{3}, 2\pi)$ D. $(0, \frac{2\pi}{3})$

27. $\int \frac{x^2-6x+8}{x-6} dx$.
A. $\frac{x^3-3x^2+8x}{(x-6)^2} + C$
B. $\frac{x^2}{2} + 8\ln|x-6| + C$
C. $\frac{x^2}{2} - \ln|x-6| + C$
D. $\frac{x^3}{3} + 6\ln|x-8| + C$

28. $\int \frac{x^2-3x}{(x-2)^3} dx$.
A. $\frac{x^3-3x^2}{6(x-2)^4} + C$
B. $\ln|x-2| - \frac{3}{x-2} - \frac{6}{(x-2)^2} + C$
C. $\ln|x-2| - \frac{1}{x-2} + \frac{1}{(x-2)^2} + C$
D. $\ln|x-2| - \frac{1}{4(x-2)^4} + \frac{1}{5(x-2)^5} + C$

29. $\int (x+2)csc(x^2+4x)cot(x^2+4x)dx$.
A. $-\frac{1}{2}csc(x^2+4x) + C$
B. $-\frac{1}{2}csc(x+2) + C$
C. $-\frac{1}{2}cot(x^2+4x) + C$
D. $-\frac{1}{2}cot(x+2) + C$

30. $\int_0^{2\pi} \int_0^1 \int_0^{\sqrt{1-r^2}} zrdzdrd\theta$.
A. $\frac{\pi}{2}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{8}$ D. $\frac{\pi}{3}$

31. Find the point on the plane $x + 2y + z = 1$ that is closest to the origin.
A. $(\frac{1}{3}, \frac{1}{6}, \frac{1}{3})$ B. $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ C. $(\frac{1}{2}, \frac{1}{6}, \frac{1}{6})$ D. $(\frac{1}{6}, \frac{1}{3}, \frac{1}{6})$

32. Suppose that the position vector of a particle moving in the plane is $r = 12\sqrt{t}\mathbf{i} + t^{\frac{3}{2}}\mathbf{j}, t > 0$. Find the minimum speed of the particle.
A. $2\sqrt{3}$ B. $3\sqrt{2}$ C. $2\sqrt{2}$ D. $3\sqrt{3}$

33. Find the arc length of the parametric curve $x = e^t, y = e^{-t}, z = \sqrt{2}t, 0 \leq t \leq 1$.
A. $\frac{e^2+1}{e}$ B. e^2 C. $e^2 + 1$ D. $\frac{e^2-1}{e}$

34. Find the region enclosed by the rose $r = 4cos(3\theta)$.

A. 8π B. 4π C. 6π D. 2π

35. Find the critical values of $f(x) = x^2 ln|x^2|$

A. $0, \sqrt{e^{-1}}, -\sqrt{e^{-1}}$ B. 0 C. $1, -1$ D. $\sqrt{e^{-1}}, -\sqrt{e^{-1}}$

36. Find the derivative of $f(x) = ln[sec^{\frac{1}{3}}(x)]$

A. $\frac{tan(x)}{3}$ B. $\frac{cot(x)}{3}$ C. $\frac{csc(x)}{3}$ D. $\frac{sin(x)}{3}$

37. Find $\int \frac{x^{\frac{5}{6}} - 4x + x^{\frac{2}{3}}}{\sqrt[3]{x^2}} dx$

A. $\frac{6}{7}x^{\frac{7}{6}} + 3x^{\frac{4}{3}} - x + C$ B. $\frac{7}{6}x^{\frac{6}{7}} - 4x^{\frac{3}{4}} + x + C$ C. $\frac{6}{7}x^{\frac{7}{6}} - 3x^{\frac{4}{3}} + x + C$ D. $\frac{7}{6}x^{\frac{6}{7}} + 4x^{\frac{3}{4}} - x + C$

38. Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(3\left(\frac{2}{n}i\right)^2 - \left(\frac{2}{n}i\right) + 1 \right) \left(\frac{2}{n}\right) \right)$

A. 6 B. 4 C. 10 D. 8

39. Find the equation of a line that is tangent to the function $y = e^x$ and passes through the origin.

A. $y = x$ B. $y = 2x$ C. $y = ex$ D. $y = ln(2)x$

40. A particle's postion is given by the function $P(t) = sin(t)cos(t)$. Determine the interval(s) where the particle is slowing down on the interval $(0, \pi)$.

A. $(0, \frac{\pi}{2})$ B. $(\frac{\pi}{4}, \frac{\pi}{2}) \cup (\frac{3\pi}{4}, \pi)$ C. $(0, \frac{\pi}{4}) \cup (\frac{\pi}{2}, \frac{3\pi}{4})$ D. $(0, \frac{\pi}{4})$